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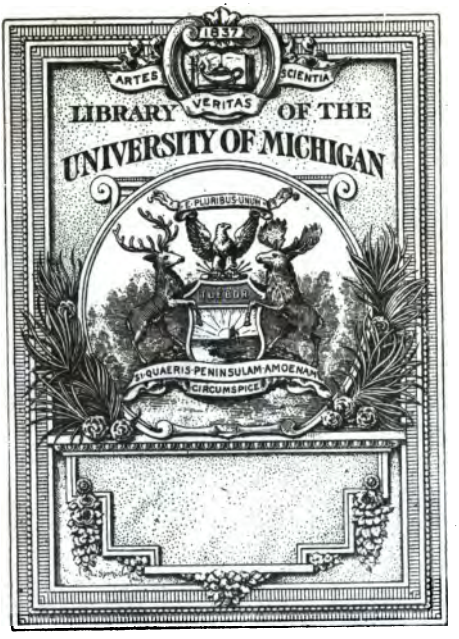
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TRIGONOMETRIÆ

PLANÆ ET SPHÆRICÆ

ELEMENTA.

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TRIGONOMETRIÆ PLANÆ ELEMENTA.

DEFINITIONES.

Definitio I.

Ex datis trianguli lateribus angulos, et ex angulis latera laterumve rationes, et mixtim assequi, Trigonometriæ munus est. Ad quod præstandum, necesse est, ut non tantum peripheriæ circulares, sed et rectæ lineæ circulis adscriptæ, in notas aliquot et certas partes secari supponantur.

II.

Placuit itaque veteribus Mathematicis, peripheriam circuli in 360 partes (quos gradus appellant) dividere; et unumquemque gradum in 60 minuta prima, et hæc singula in 60 secunda, et rursus horum unumquodque in 60 minuta tertia, et ita continuo partiri. Et angulus quilibet dicitur esse tot graduum et minutorum, quot sunt in arcu qui angulum illum metitur.

Quidam gradum in partes centesimas, potius quam sexagesimas partiri volunt: et utilius fortasse esset, non gradus sed et ipsum circulum in decupla ratione secare; quæ divisio forsan aliquando obtinebit. Verum si circulus constet 360 gradibus, ejus quadrans quæ est mensura anguli recti, erit harum partium 90. Si circulus in 100 partes secetur, quadrans erit 25 partium.

III.

Complementum arcus, eſt differentia ejus a quadrante.

IV.

Chorda ſive *ſubtenſa* eſt recta linea ab uno arcus termino ad alterum ducta.

V.

Sinus rectus alicujus arcus, qui et ſimpliciter ſinus dici ſolet, eſt perpendicularis cadens ab uno arcûs termino ad diametrum per alterum terminum ejuſdem arcûs ductum.

Corol. 1. Eſt igitur ſinus rectus alicujus arcus ſemiſubtenſa dupli arcus: ſcilicet eſt D E = ½ D O, et eſt arcus D O duplus ipſius D B. *Vide Fig.* 1. *Tab.* 1.

Corol. 2. Hinc ſinus arcus triginta graduum æqualis eſt dimidio radii, nam per 15 El. iv. Latus Hexagoni circulo inſcripti, hoc eſt, ſubtenſa ſexaginta graduum æqualis eſt radio.

VI.

Sinus dividit radium in duo ſegmenta C E E B; quorum unum C E quod centro et ſinu recto intercipitur, eſt ſinus complementi arcus D B ad quadrantem, (nam eſt C E = F D qui eſt ſinus arcus D H,) et vocatur *coſinus*.

VII.

Alterum ſegmentum B E, quod ſinu recto et peripheria intercipitur, vocatur *ſinus verſus*: aliquando dicitur arcus *ſagitta*.

VIII.

Quod ſi per unum arcus terminum D producatur a centro recta C G, donec occurrat rectæ B G ſuper diametro ad ejus terminum B perpendiculari; vocabitur in Trigonometria C G *ſecans*, et B G *tangens* arcus D B.

IX.

Coſecans et cotangens arcus eſt ſecans vel tangens arcus, qui eſt complementum alterius ad quadrantem.

Nota. Sicut eadem eſt chorda arcus et ejuſdem complementi ad circulum, ſic idem eſt ſinus, eadem tangens, eademque ſecans arcus et ejuſdem complementi ad ſemicirculum.

X. Sinus

X.

Sinus totus eſt ſinus maximus, ſeu ſinus nonaginta graduum qui circuli radio æqualis eſt.

XI.

Canon Trigonometricus eſt tabula, quæ a minuto incipiens, ſeriatim exhibet quas habent longitudines ſinguli ſinus tangentes et ſecantes, reſpectu radii, qui unitatis loco ponitur, et in partes 10 000 000 vel plures decimales dividi intelligitur. Adeo ut ope hujus tabulæ, cujuſlibet arcus vel anguli ſinus tangens vel ſecans haberi poteſt; et viciſſim ex dato ſinu tangente vel ſecante dabitur qui iis reſpondet arcus vel angulus. Obſervandum eſt in ſequentibus

R. eſſe notam radii, S. notam ſinus,
T. notam tangentis, cof. notam cofinus,
cot. notam cotangentis.

Corol. *Vide figuram Definitionum.*

CE : ED :: CB : BG. hoc eſt, cof. : S. :: R. : T.
GB : BC :: CH : HI. h. e. T. : R. :: R. : cot.
CE : CD :: CB : CG. h. e. cof. : R. :: R. : ſecant.
DE : CD :: CH : CI. h. e. S. : R. :: R. : coſecant.

CONSTRUCTIO
CANONIS TRIGONOMETRICI.

PROP. I. THEOREMA.

Datis duobus quibuſlibet trianguli rectanguli lateribus, reliquum quoque dabitur.

Eſt enim per 47 Elementi primi $AC^2 = AB^2 + BC^2$ et $AC^2 - BC^2 = AB^2$, et viciſſim $AC^2 - AB^2 = BC^2$. Unde per extractionem radicis quadratæ, dabitur $AC = \sqrt{AB^2 + BC^2}$ et $AB = \sqrt{AC^2 - BC^2}$. et $BC = \sqrt{AC^2 - AB^2}$.

PROP. II. PROBL.

Dato D E *ſinu arcus* D B, *invenire coſinum* D F.

Ex datis C D radio et D E ſinu, in triangulo rectangulo C D E dabitur per præcedentem $CE = \sqrt{CD^2 - DE^2} = DF$. *Hoc eſt* cof. $= \sqrt{R^2 - s^2}$.

PROP. III. PROBL.

Dato D E *ſinu arcus cujuſvis* D B, *invenire* D M *vel* B M *ſinum arcus dimidii.*

Dato D E dabitur per præcedentem C E, ac proinde E B, quæ eſt differentia inter cofinum et radium. In triangulo igitur rectangulo D B E datis D E et E B dabitur D B cujus ſemiſſis D M eſt ſinus arcus D L $= \frac{1}{2}$ arcus D B. *Hoc eſt* $s.\frac{A}{2} = \frac{\sqrt{s^2 A + \text{verſ. } s^2 A}}{2}$.

PROP. IV. PROBL.

Dato B M *ſinu arcus* B L, *invenire ſinum dupli arcus.*

Dato B M ſinu, dabitur per Prop. ii. cofinus C M. Sunt autem triangula C B M D B E æquiangula, ob angulos ad E et M rectos et angulum ad B communem, quare (per IV. 6.) erit C B : C M :: B D vel 2 B M : D E. *Hoc eſt* R. : cof. :: 2 s. : s. dupli arcus. Unde cum dantur tres priores hujus analogiæ termini, quartus quoque qui eſt ſinus arcus D B innoteſcet. *Hoc eſt* $s.2A = \frac{2\,s.A \times \text{cof. } A}{R.}$.

Corol. Eſt C B : 2 C M :: B D : 2 D E, hoc eſt, radius ad duplum cofinus arcus $\frac{1}{2}$ D B ut ſubtenſa arcus D B ad ſubtenſam dupli arcus. Item eſt C B : 2 C M :: (2 B M : 2 D E : :) B M : D E : : $\frac{1}{2}$ C B : C M. (*Hoc eſt* s. : s. dupli arcus : : $\frac{R}{2}$: cof.) Unde dato ſinu arcus alicujus et ſinu arcus dupli, dabitur cofinus arcus ſimpli.

PROP.

PROP. V.

Datis ſinubus et proinde etiam coſinubus duorum arcuum B D, F D, (*quorum major ſit* B D,) *invenire ſinum et coſinum ſummæ arcuum, ut etiam ſinum et coſinum differentiæ eorundem.*

Eſto D K ſinus et C K coſinus arcus D B, fiat autem arcus D E = D F. Erit igitur B F ſumma et B E differentia arcuum B D, F D, et ductis F E et radio C D erit F E ad ipſum C D perpendicularis in puncto O. Erit itaque F O = $\frac{F E}{2}$ ſinus et C O coſinus arcus F D. Per F et E ducantur F I et E L ipſi D K parallelæ, erit igitur F I ſinus et C I coſinus arcus B F; E L autem ſinus et C L coſinus arcus B E: agatur etiam per O, O P eidem D K parallela, et per O et E agantur ipſi C B parallelæ M O et etiam G E occurrens ipſi P O in N; itaque ob parallelas M O, G E, et F O = O E erit F M = M G, hoc eſt ob F I et O P parallelas = O N. Item ob F O = $\frac{F E}{2}$ erit M O ſive G N = $\frac{G E}{2}$ et proinde G N ſive I P = N E ſive P L; erit itaque F I (ſin. ſumm.) = O P + F M et E L (ſin. diff.) = O P − O N ſive F M. Erit etiam C I = C P − P I ſive M O, et C L = C P + P L ſive M O.

Patet porro triangula C D K, C O P, F O H, F O M æquiangula eſſe.

Erit ergo C D : D K :: C O : O P.

Hoc eſt

$$R : S.DB :: \text{cof.}FD : \frac{S.DB \times \text{cof.}FD}{R} = OP.$$

Item C D : C K :: F O : F M.

Hoc eſt

$$R : \text{cof.}DB :: S.FD : \frac{\text{cof.}DB \times S.FD}{R} = FM.$$

1. Itaque F I (ſin. ſumm.)

$$= OP + FM = \frac{S.DB \times \text{cof.}FD + \text{cof.}DB \times S.FD}{R}.$$

2. Et E L (ſin. diff.)

$$= OP - FM = \frac{S.DB \times \text{cof.}FD - \text{cof.}DB \times S.FD}{R}.$$

Erit etiam C D : C K : : C O : C P.
Hoc eſt

$$R. : \text{cof. D B} :: \text{cof. F D} : \frac{\text{cof. D B} \times \text{cof. F D}}{R} = \text{C P.}$$

Item C D : D K : : F O : M O.
Hoc eſt

$$R. : \text{S. D B} :: \text{S. F D} : \frac{\text{S. D B} \times \text{S. F D}}{R} = \text{M O} = \text{I P vel P L.}$$

3. Itaque C I (cos. ſumm.)

$$= \text{C P} - \text{P I} = \frac{\overline{\text{cof. D B} \times \text{cof. F D}} - \overline{\text{S. D B} \times \text{S. F D}}}{R}.$$

4. Et C L (cof. diff.)

$$= \text{C P} + \text{P L vel P I} = \frac{\overline{\text{cof. D B} \times \text{cof. F D}} + \overline{\text{S. D B} \times \text{S. F D}}}{R}.$$

Coroll. Quia arcuum B E, B D, B F, differentiæ ſunt æquales, erit B D arcus medius arithmeticus inter arcus B E, B F.

Hinc dato ſinu et coſinu arcus alicujus, facile invenitur ſinus et coſinus dupli arcus.

Scribatur, compendii gratia, A *pro arcu* D B et B *pro arcu* F D. *Itaque formulæ iſtæ* 4 *ſupra demonſtratæ hunc in modum ſcribendæ erunt.*

1. $\text{S.}\overline{\text{A} + \text{B}} = \frac{\overline{\text{S. A} \times \text{cof. B}} + \overline{\text{cof. A} \times \text{S. B}}}{R}.$

2. $\text{S.}\overline{\text{A} - \text{B}} = \frac{\overline{\text{S. A} \times \text{cof. B}} - \overline{\text{cof. A} \times \text{S. B}}}{R}.$

3. $\text{cof.}\overline{\text{A} + \text{B}} = \frac{\overline{\text{cof. A} \times \text{cof. B}} - \overline{\text{S. A} \times \text{S. B}}}{R},$

4. $\text{cof.}\overline{\text{A} - \text{B}} = \frac{\overline{\text{cof. A} \times \text{cof. B}} + \overline{\text{S. A} \times \text{S. B}}}{R}.$

Jam ſi A *ponatur ipſi* B *æqualis erit* A + B = 2 A, *et* S. *vel* cof. A = S. *vel* cof. B. *Unde in nihilum abit æquatio* 2*da*, 4*ta vero fiat* cof. A − B = R. *Prima denique et tertia fient*

5. $\text{S. 2 A} = \frac{2 \times \text{S. A} \times \text{cof. A}}{R}.$

6. cof.

6. $\text{cof. } 2\,A = \frac{\text{cof.}^2 A - s^2 A}{R}$.

Æquatione autem quinta in proportionem verſa erit R : cof. A : : 2 s. A : s. 2 A (*quæ eſt bujuſce Prop. iv.*) *vel* R. : 2 cof. A : : s. A : s. 2 A (*quæ eſt Coroll. Prop. iv. pars* 1.) *Porro ex* 2 s. A : s. 2 A : : R. : cof. A *erit* s. A : s. 2 A : : $\frac{R}{2}$: cof. A (*ejuſdem Coroll. pars* 2.)

Etiam æq. ſexta in proportionem facile mutatur; cum enim $\text{cof. } 2\,A = \frac{\text{cof.}^2 A - \text{s.}^2 A}{R} = \frac{\overline{\text{cof. A} + \text{s. A}} \times \overline{\text{cof. A} - \text{s. A}}}{R}$ *erit* R. : cof. A + s. A : : cof. A − s. A : cof. 2 A.

PROP. VI.

Si ſint tres arcus arithmetice proportionales, erit radius ad duplum coſinús arcús medii ut ſinus differentiæ ad differentiam ſinuum extremorum.

Nam hi tres arcus per B E, B D, B F, ſemper exponi poſſunt, (vel quod tantundem eſt, trium magnitudinum arithmetice proportionalium minima æqualis eſt ſemper mediæ ablatâ communi differentiâ, maxima vero æqualis eidem mediæ communi differentiâ adjectâ,) et ſinus horum arcuum ſunt $\frac{\text{s. B D} \times \text{cof. F D} + \text{cof. B D} \times \text{s. F D}}{R}$, s. B D, et $\frac{\text{s. B D} \times \text{cof. F D} - \text{cof. B D} \times \text{s. F D}}{R}$. Differentia igitur ſinuum extremorum hoc eſt s. B F − s. B E = $\frac{2 \times \text{cof. B D} \times \text{s. F D}}{R}$: erit itaque R. : 2 cof. B D : : s. F D : s. B F − s. B E. *Q. E. D.*

Cor. 1. Si arcus B D ſit 60 grad. erit differentia ſinuum F I F L æqualis F O ſinui diſtantiæ. Nam in eo caſu erit C K (coſinus ſc. 60 grad.) ſinus 30 grad. cujus duplum æquale eſt radio, adeoque ob C D rad. = 2 C K erit F O ſin. diff. = F G differentiæ ſinuum ext. Adeoque ſi duo arcus B E B F ab arcu 60 gr. æquidiſtent, erit differentia ſinuum æqualis ſinui diſtantiæ F D.

Cor. 2. Hinc ſi dentur ſinus omnium arcuum, dato intervallo a ſe invicem diſtantium ab initio quadrantis uſque

uſque ad 60 gradus, facile inveniuntur reliqui per unicam additionem. Eſt enim ſinus 61 gr. = ſinui 59 gr. + ſin. 1 gr. et ſinus 62 gr. = ſinui 58 gr. + ſin. 2 gr. Item ſinus 63 gr. = ſinui 57 gr. + ſin. 3 gr. et ita deinceps.

Cor. 3. Si habeantur ſinus omnium arcuum ab initio quadrantis, dato intervallo a ſe invicem diſtantium, uſque ad datam quamvis quadrantis partem, dabuntur exinde ſinus omnes uſque ad hujus partis duplum. *ex. g.* Dentur omnes ſinus uſque ad 15 gr. per præcedentem analogiam inveniri poſſunt ſinus omnes uſque ad 30 gr. Nam eſt radius ad duplum coſinus 15 gr. (vel arcûs medii) ut ſinus unius gradus ad differentiam ſinuum 14 gr. et 16 gr. ita etiam eſt ſinus 2 gr. ad differentiam ſinuum 13 et 17 gr. et ita ſinus 3 gr. ad differentiam ſinuum 12 et 18 gr. atque ſic continuo uſque dum pervenietur ad ſinum 30 gr.

Similiter ut radius ad duplum coſinus 30 gr. ſeu ad duplum ſinus 60 gr. ita ſinus 1 gr. ad differentiam ſinuum 29 et 31 gr. : : ſin 2 gr. ad differentiam ſinuum 28 et 32 gr. :: ſin 3 gr. ad differentiam ſinuum 27 et 33 gr. ſed in hoc caſu eſt [a] radius ad duplum coſinus 30 gr. ut 1 ad $\sqrt{3}$. ac proinde ſi multiplicentur ſinus diſtantiarum ab arcu 30 gr. per $\sqrt{3}$ dabuntur differentiæ ſinuum.

Similiter in ipſo initio quadrantis minutim exquirere poſſumus ſinus, datis ſinubus et coſinubus unius et duorum minutorum. Nam ut radius ad duplum coſinus 2′ : : ſin 1′ : differentiam ſinuum 1′ et 3′ : Sin 2′ : differentiam ſinuum 0′ et 4′ hoc eſt, ad ipſum ſinum 4′. Et ſimiliter ex datis ſinubus priorum 4′ inveniuntur ſinus reliqui uſque ad 8′ et exinde ad 16′ et ita deinceps [b].

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[a] *Sit* D B *arcus* 30 *gr.* C D = *rad.* = 1 *erit duplum* D K = $\frac{1}{2}$. *Deinde* $\sqrt{\text{C D q} - \text{D K q}} = \text{C K} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}}$. *Sed* (*per* 14*tam* 5*ti*) C D : C K : : 1 : $\sqrt{\frac{3}{4}}$ *et duplicando conſequentes* C D (*rad.*) : 2 C K (*dupl.* coſ. 30 *gr.*) : : 1 : 2 $\sqrt{\frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$. *Vide fig. Prop.* v.

[b] *Ex his tamen Corollariis* 1 *et* 2 *abſque ope hujus propoſitionis*

PROP. VII. THEOREMA.

In arcubus exiguis ſinus et tangens ejuſdem arcus ſunt quam proxime ad ſe invicem, in ratione æqualitatis.

Nam ob æquiangula triangula C E D C B G, erit C E : C B :: E D : B G. Sed accedente puncto D ad B, eva-

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poſitionis demonſtrari poſſunt. Æquatio enim ſecunda a prima ſubducatur, et prodibit $\text{S.A} + \text{B} - \text{S.A} - \text{B} = \frac{2\,\text{cof.A} \times \text{S.B}}{\text{R}}$. *Sit jam* $\text{A} = 60°$ *et erit* $2\,\text{cof.A} = \text{R}$. *Igitur* $\text{S.}\overline{60 + \text{B}} - \text{S.}\overline{60 - \text{B}} = \text{S.B}$. *Et igitur* $\text{S.}\overline{60 + \text{B}} = \text{S.B} + \text{S.}\overline{60 - \text{B}}$. *Itaque poſito* $\text{B} = 1°, 2°, 3°$, *&c. erit* $\text{S.}61° = \text{S.}1° + \text{S.}59°$, $\text{S.}62° = \text{S.}2° + \text{S.}58°$, *&c. Itaque datis ſinubus et coſinubus uſque ad* 60° *cætera per ſolam additionem derivari poſſunt.*

Utile etiam eſt hoc theorema, quod a formulis præcedentibus derivari poteſt, nempe quod $\text{S.}\overline{30 + \text{B}} = \text{cof.B} - \text{ſin.}\overline{30 - \text{B}}$. *Nam ex additione æq.* 1 *et* 2 *prodibit*

$$\text{S.}\overline{\text{A} + \text{B}} + \text{S.}\overline{\text{A} - \text{B}} = \frac{2\,\text{S.A} \times \text{cof.B}}{\text{R}}.$$

Sit jam A 30 *gr. itaque* $2\,\text{S.A} = \text{R}$. *quocirca erit*

$$\text{S.}\overline{30 + \text{B}} + \text{S.}\overline{30 - \text{B}} = \text{cof.B}.$$

Et $\text{S.}\overline{30 + \text{B}} = \text{cof.B} - \text{S.}\overline{30 - \text{B}}$. *itaque poſito* $\text{B} = 1°, 2°, 30°$, *&c. erit* $\text{S.}31° = \text{cof.}1° - \text{S.}29°$. $\text{S.}32° = \text{cof.}2° - \text{S.}28°$, $\text{S.}33° = \text{cof.}3° - \text{S.}27°$ &c. *Quare datis ſinubus et coſinubus arcuum uſque ad* 30° *cæteri uſque ad* 60° *per ſolam ſubtractionem reperientur : a* 60° *autem ad* 90° *per ſolam additionem ut ſupra.*

De calculo etiam tangentium qui interdum apprime utilis eſt pauca adjicienda ſunt.

Datis ſinubus et coſinubus, tangentibus et cotangentibus duorum arcuum, invenire tang. et cotang. ſummæ et differentiæ iſtorum arcuum.

Arcus cujuſlibet ut notum eſt $\text{T.} = \frac{\text{R} \times \text{S}}{\text{cof.}}$, *quare eadem, quæ ſupra, adhibita notatione,* $\text{T.}\overline{\text{A} + \text{B}} = \frac{\text{R.} \times \text{S.}\overline{\text{A} + \text{B}}}{\text{cof.A} + \text{B.}}$.

Hoc

neſcit E B reſpectu arcus B D : unde fit C E fere æqualis C B. adeoque et E D fere æqualis B G. Si E B fit minor radii parte $\frac{1}{10\,000\,000}$ erit differentia inter ſinum et tangentem, minor quoque tangentis parte $\frac{1}{10\,000\,000}$.

Hoc eſt (*ex æq.* 1. *et* 3.)

$$\text{T.A} + \text{B} = \frac{\text{R} \times \overline{\text{S.A} \times \text{cof.B} + \text{S.B} \times \text{cof.A}}}{\overline{\text{cof.A} \times \text{cof.B} - \text{S.A} \times \text{S.B}}},$$ *et æq. in proportionem verſa.*

$$\overline{\text{cof.A} \times \text{cof.B} - \text{S.A} \times \text{S.B}} : \overline{\text{S.A} \times \text{cof.B} + \text{S.B} \times \text{cof.A}} :: \text{R.} : \text{T.A} + \text{B}$$ *et diviſis prioribus duobus terminis per* cof. A × cof. B *prodibit*

$$1 - \frac{\text{S.A} \times \text{S.B}}{\text{cof.A} \times \text{cof.B}} : \frac{\text{S.A}}{\text{cof.A}} + \frac{\text{S.B}}{\text{cof.B}} :: \text{R.} : \text{T.}\overline{\text{A} + \text{B}},$$

et ſcribendo $\frac{\text{T}}{\text{R}}$ *pro* $\frac{\text{S}}{\text{cof.}}$ *erit*

$$1 - \frac{\text{T.A}}{\text{R}} \times \frac{\text{T.B}}{\text{R}} \text{ ſive } \frac{\text{R}^2 - \overline{\text{T.A} \times \text{T.B}}}{\text{R}^2} : \frac{\text{T.A} + \text{T.B}}{\text{R}}$$

$:: \text{R.} : \text{T.}\overline{\text{A} + \text{B}}$, *quocirca* $\text{T.}\overline{\text{A} + \text{B}} = \frac{\text{R}^2 \times \overline{\text{T.A} + \text{T.B}}}{\text{R}^2 - \overline{\text{T.A} \times \text{T.B}}}$, *et ſi ponatur* A = B *erit* $\text{T.2A} = \frac{\text{R}^2 \times 2\,\text{T.A}}{\text{R}^2 - \overline{\text{T.A}^2}}$.

Cotangens autem arcus cujuſlibet $= \frac{\text{R}^2}{\text{T}}$, *quocirca erit*

Cotang. $\text{A} + \text{B} = \frac{\text{R}^2 - \overline{\text{T.A} \times \text{T.B}}}{\text{T.A} + \text{T.B}}$, *et ſi* A = B *erit cotang.* $2\text{A} = \frac{\text{R}^2 - \overline{\text{T.A}^2}}{2\,\text{T.A}}$.

Simili ratione erit $\text{T.A} - \text{B} = \frac{\text{R}^2 \times \overline{\text{T.A} - \text{T.B}}}{\text{R}^2 + \overline{\text{T.A} \times \text{T.B}}}$, *et etiam cotang.* $\text{A} - \text{B} = \frac{\text{R}^2 + \text{T.A} \times \text{T.B}}{\text{T.A} - \text{T.B}}$.

Cor.

Cor. Cum arcus ſit tangente minor, ſinu autem ſuo major; et exigui arcus ſinus et tangens ſunt fere æquales, erit etiam arcus ſuo ſinui vel tangenti fere æqualis, adeoque in exiguis arcubus, erit ut arcus ad arcum ita ſinus ad ſinum.

PROP. VIII.

Invenire ſinum arcus unius minuti.

Latus hexagoni circulo inſcripti, hoc eſt, ſubtenſa 60 graduum æqualis eſt radio, (per 15tam 4ti.) Radii itaque ſemiſſis erit ſinus arcus 30 gr. Dato itaque ſinu arcus 30 grad. invenitur ſinus arcus 15 gr. (per 3tiam hujus.) Item ex dato ſinu 15 gr. per eandem invenitur ſinus 7 gr. 30 min. et ſinus hujus dimidii 3 gr. 45′ ſimiliter invenitur: et ita deinceps, donec duodecima peracta biſectione, perveniatur ad arcum $52'' 44''' 3'''' 45''''' 0''''''$ cujus coſinus fere æqualis eſt radio, in quo caſu (uti conſtat ex Prop. vii.) ſunt ſinus arcubus ſuis proportionales; adeoque ut arcus $52'' 44''' 3'''' 45''''' 0''''''$ ad arcum unius minuti ita erit ſinus prius inventus ad ſinum arcus unius minuti, qui igitur dabitur.

Dato ſinu unius minuti, invenietur per Prop. ii. et iv. ſinus duorum minutorum ejuſque coſinus.

PROP. IX. THEOREMA.

Si angulus B A C *in peripheria circuli exiſtens, biſecetur recta* A D, *et producatur* A C *quoad* D E = A D *ipſi occurrat in* E: *erit* C E = A B.

In quadrilatero A B C D (per xxii. 3.) ſunt anguli B et A C D æquales duobus rectis = D C E + D C A (per xiii. 1.) unde erit angulus B = D C E. Quin etiam eſt angulus E = D A C (per v. 1.) = D A B et eſt D C = D B. Quare triangula B A D et C E D ſunt congrua, et C E eſt æqualis A B. *Q. E. D.*

PROP.

PROP. X. THEOREMA.

Sint arcus A B B C C D D E E F *&c. æquales; arcuimque* A B A C A D A E *&c. ſubtenſæ ducantur, erit* A B : A C : : A C : A B + A D : : A D : A C + A E : : A E : A D + A F : A F : A E + A G.

Producantur A D in H, A E in I, A F in K, et A G in L, ut triangula A C H A D I A E K A F L ſint Iſoſcelia. Et quoniam angulus B A D biſectus eſt, fiet D H = A B per præcedentem. Similiter erit E I = A C, F K = A D, item G L = A E.

Sed triangula Iſoſcelia A B C C A H D A I E A K F A L, ob angulos ad baſes æquales, ſunt æquiangula. Quare erit ut A B : A C : : A C : A H = A B + A D : : A D : A I = A C + A E : : A E : A K = A D + A F : : A F : A L = A E + A G. *Q. E. D.*

Corol. Quoniam eſt A B ad A C ut radius ad duplum coſinus arcus $\frac{1}{2}$ A B, (per. Corol. Prop. iv.) erit quoque ut radius ad duplum coſinus arcus $\frac{1}{2}$ A B ita $\frac{1}{2}$ A B : $\frac{1}{2}$ A C : : $\frac{1}{2}$ A C : $\frac{1}{2}$ A B + $\frac{1}{2}$ A D : : $\frac{1}{2}$ A D : $\frac{1}{2}$ A C + $\frac{1}{2}$ A E : : $\frac{1}{2}$ A E : $\frac{1}{2}$ A D + $\frac{1}{2}$ A F &c. Sint jam arcus A B B C C D &c. ſinguli 2′. Erit $\frac{1}{2}$ A B ſinus unius minuti, $\frac{1}{2}$ A C ſinus 2′ $\frac{1}{2}$ A D ſinus 3′ $\frac{1}{2}$ A F ſinus 4′ &c. Unde datis ſinubus unius et duorum minutorum ſinus omnes reliqui ſic facillime habentur.

Dicatur coſinus arcus unius minuti, hoc eſt, ſinus arcus 89 gr. 59′ a. et fient ſequentes analogiæ, R : 2 a : : ſin 2′ : s 1′ + s 3′. Quare dabitur ſinus 3′. Item R : 2 a : : s. 3′ : s. 2′ + s. 4′. quare dabitur s 4′.

Item R : 2 a : : s. 4′ : s. 3′ + s. 5′. quare habetur ſinus 5′.

R : 2 a : : s. 5′ : s 4′ + s 6′ proinde dabitur s 6′. Atque ita deinceps ad ſingula quadrantis minuta dabuntur ſinus. Et quoniam radius ſeu primus analogiæ terminus eſt unitas; operationes per multiplicationem contractam et ſubductionem facillime expediuntur.

Inventis ſinubus, uſque ad gradum ſexageſimum. Reliqui ſinus per ſolam additionem habentur (per Cor. 2. Prop. 6.).

Datis ſinubus, tangentes et ſecantes ex analogiis inveniri poſſunt, (in fig. Definitionum) ob æquiangula triangula C E D C B G C H I.

SCHO-

SCHOLIUM.

Magnus ille Geometra, ſummuſque Philoſophus Dominus Newtonus primus ſeries in infinitum convergentes exhibuit, quibus ex datis arcubus, eorum ſinus computari poſſint. Nam ſi arcus dicatur A *et radius ſit unitas, invenit ejus ſinum fore.*

$$A - \frac{A^3}{1.2.3} + \frac{A^5}{1.2.3.4.5} - \frac{A^7}{1.2.3.4.5.6.7} + \frac{A^9}{1.2.3.4.5.6.7.8.9}$$

&c. Coſinum autem eſſe

$$1 - \frac{A^2}{1.2} + \frac{A^4}{1.2.3.4} - \frac{A^6}{1.2.3.4.5.6} + \frac{A^8}{1.2.3.4.5.6.7.8} \text{ \&c.}$$

Hæ ſeries initio quadrantis cum arcus A *parvus eſt celerrime convergunt. Nam in ſerie pro ſinu, ſi* A *non ſuperet decem minuta, duo primi ejus termini ſcil.* $A - \frac{1}{6} A^3$ *dant ſinum ad* 15 *figurarum loca, ſi arcus* A *non major ſit gradu, tres primi exhibent ſinum ad totidem loca, adeoque pro primis et ultimis quadrantis ſinubus hæ ſeries ſunt admodum utiles: ſed quo major ſit arcus* A, *eo pluribus opus eſt terminis ut inveniatur ſinus in numeris qui ſunt veri ad datum figurarum locum. Tandem autem lentiſſime convergunt ſeries cum arcus fere æqualis eſt radio. Cui rei ut remedium adferatur, ego alias excogitavi ſeries Newtonianis ſimiles, in quibus ſuppono arcum cujus ſinus quæritur eſſe ſummam vel differentiam duorum arcuum ſcil. eſſe* A + z *vel* A − z: *notoſque eſſe ſinum et coſinum arcus* A. *ſcil. ſit* a *ſinus arcus* A *et* b *ejus coſinus. Sinus arcus* A + z *per hanc ſeriem exprimetur*

1. $a + \frac{bz}{1} - \frac{az^2}{1.2} - \frac{bz^3}{1.2.3} + \frac{az^4}{1.2.3.4} + \frac{bz^5}{1.2.3.4.5}$ &c.

2. *Ejus coſinus* $b - \frac{az}{1} - \frac{bz^2}{1.2} + \frac{az^3}{1.2.3} + \frac{bz^4}{1.2.3.4} - \frac{az^5}{1.2.3.4.5} - \frac{bz^6}{1.2.3.4.5.6}$ *&c.*

Similiter ſinus arcus A − z *eſt*

3. $a - \frac{bz}{1} - \frac{az^2}{1.2} + \frac{bz^3}{1.2.3} + \frac{az^4}{1.2.3.4} - \frac{bz^5}{1.2.3.4.5} - \frac{az^6}{1.2.3.4.5.6}$ *&c.*

Et

Et cosinus est

4. $b + \frac{az}{1} - \frac{bz^2}{1.2} - \frac{az^3}{1.2.3} + \frac{bz^4}{1.2.3.4} + \frac{az^5}{1.2.3.4.5}$ &c.

Arcus A *est medius arithmeticus inter arcus* A − z *et* A + z. *Differentiæ sinuum sunt*

5. $\frac{bz}{1} - \frac{az^2}{1.2} - \frac{bz^3}{1.2.3} + \frac{az^4}{1.2.3.4} + \frac{bz^5}{1.2.3.4.5} - \frac{az^6}{1.2.3.4.5.6}$ &c.

6. $\frac{bz}{1} + \frac{az^2}{1.2} - \frac{bz^3}{1.2.3} - \frac{az^4}{1.2.3.4} + \frac{bz^5}{1.2.3.4.5} + \frac{az^6}{1.2.3.4.5.6}$ &c.

Unde differentiarum differentia, seu differentia secunda

7. *Prodit* $\frac{2az^2}{1.2} - \frac{2az^4}{1.2.3.4} + \frac{2az^6}{1.2.3.4.5.6}$ &c.

Seu $\frac{2a \times z^2}{1.2} - \frac{z^4}{1.2.3.4} + \frac{z^6}{1.2.3.4.5.6}$ &c.

Quæ series æqualis est duplo sinus arcus medii ducto in sinum versum arcus z *et celerrime convergit. Adeo ut si* z *sit minutum primum, terminus seriei primus dat differentiam secundam ad* 15 *figurarum loca; secundus autem terminus ad* 25 *loca.*

Hinc datis sinubus duorum quorumvis arcuum intervallo minuti distantium, facili admodum operatione inveniri possint sinus reliquorum omnium arcuum qui sunt in eadem progressione.

In serie prima et secunda si arcus A *sit* = 0, *erit* a = 0, *et* b *ejus cosinus sit radius seu* 1. *et hinc destructis terminis ubi est* a, *et pro* b *posito* 1, *series deveniunt Newtonianæ. In serie tertia et quarta, si* A *sit* 90 *gr. fiet* b = 0 *et* a = 1, *unde quoque destructis terminis ubi est* b, *et pro* a *posito* 1, *rursus prodibunt series Newtonianæ.*

Omnes hæ series ex Newtonianis facile fluunt per Prop. v. *hujus.*

PROP.

PROP. XI.

In triangulo rectangulo, ſi hypotenuſa ſit radius, latera ſunt ſinus angulorum oppoſitorum; ſi vero crus alterum fiat radius, crus reliquum eſt tangens anguli oppoſiti, et hypotenuſa eſt anguli ſecans.

Manifeſtum eſt C B eſſe ſinum arcus C D, ejuſque coſinum eſſe A B, ſed arcus C D eſt menſura anguli A, et complementum menſuræ anguli C. Præterea in ſecunda figura poſito A B radio, eſt B C, tangens et A C ſecans arcus B D, qui eſt menſura anguli A, et ſimiliter in eadem figura poſito B C radio, eſt B A tangens et A C ſecans arcus B E vel anguli C. *Q. E. D.*

Eſt igitur, ut A C ſecundum datam quamvis menſuram æſtimata ad B C in eadem menſura æſtimatam, ita erit 10000000 numerus partium in quas dividi ſupponitur radius, ad numerum qui exprimit in iiſdem partibus longitudinem quam habet ſinus anguli A, hoc eſt,

Erit A C : B C : : R : S. A
Simili ratione erit A C : B A : : R : S. C
Item A B : B C : : R : T. A
Et B C : B A : : R : T. C

In his itaque proportionalibus ſi dantur tres quælibet, per regulam trium inveniętur quarta.

PROP. XII.

Trianguli plani latera ſunt ut ſinus angulorum oppoſitorum.

Trianguli circulo inſcripti latera perpendicularibus radiis biſecentur. Et erunt ſemilatera ſinus angulorum ad peripheriam. Eſt enim angulus B D C ad centrum duplex anguli B A C ad peripheriam (per 20. El. III.) cujus itaque dimidium ſc. B D E æquale eſt B A C, atque ejus ſinus eſt B E. Eadem ratione erit B F ſinus anguli B C A. Et A G erit ſinus anguli A B C.

In triangulo rectangulo eſt B D = ½ B C = radio (per 31. El. III.) ſed radius eſt ſinus anguli recti, unde ½ B C eſt ſinus anguli A.

In triangulo amblygonio, ductis B L C L, erit angulus L complementum anguli A ad duos rectos (per 22.

El. III.) ac proinde idem erit utriuſque anguli ſinus. Eſt autem BDE (cujus ſinus eſt BE) = angulo L. quare erit et BE ſinus anguli BAC. Sunt itaque in omni triangulo ſemiſſes laterum ſinus angulorum oppoſitorum, manifeſtum autem eſt latera eſſe inter ſe ut ipſorum ſemiſſes. *Q. E. D.*

PROP. XIII.

In triangulo plano ſumma crurum, differentia crurum, tangens ſemiſummæ angulorum ad baſim, et tangens ſemidifferentiæ eorundem ſunt proportionales.

Sit triangulum ABC cujus crura AB BC et baſis AC. producatur AB ad H ut fit BH = BC. Erit AH ſumma crurum; fiat BI = BA, et erit IH differentia crurum. Item eſt HBC angulus = angulis A + ACB (per 32. El. I.) cujus itaque dimidium EBC = ſemiſummæ angulorum A et ACB, ejuſque tangens (poſito radio = EB) eſt EC. Ducatur BD ad AC parallela, fiatque HF = CD. Et ob HB = CB erit (per 4. El. I.) angulus HBF = CBD = BCA. (per 29. El. I.) Eſt etiam angulus HBD = angulo A. unde erit FBD differentia angulorum A et ACB: et EBD eorum ſemidifferentia, cujus tangens eſt ED. Per I ducatur IG parallela ad AC vel BD, et fiet (per 2. El. VI.) AB: BI::CD:DG. At eſt AB = BI, unde erit et CD = DG. At eſt CD = HF, unde HF = DG et proinde HG = DF et ½HG = ½DF = DE. Et quia triangula AHC IHG ſunt æquiangula, erit AH:IH:: HC:HG::½HC:½HG::EC:ED. Hoc eſt, erit AH ſumma crurum ad IH differentiam crurum, ut EC tangens ſemiſſis ſummæ angulorum ad baſim, ad ED tangentem ſemiſſis differentiæ eorundem. *Q. E. D.*

PROP. XIV.

In triangulo plano, baſis, ſumma laterum, differentia laterum, differentia ſegmentorum baſis ſunt proportionales.

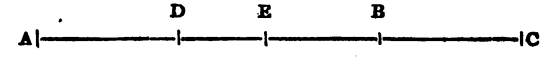
Trianguli BCD baſis eſto DC, centro B radio BC deſcribatur circulus, et producatur DB in G, ex puncto B in baſin cadat perpendicularis BE, erit DG = DB +

BC

B C = ſummæ laterum, et D H = differentiæ laterum, et ſegmenta baſis ſunt D E C E quorum differentia eſt D F. Quoniam (per Cor. Prop. xxxvi. El. III.) rectangulum ſub D C D F æquale eſt rectangulo ſub D G D H, erit (per 16. El. VI.) D C : D G : : D H : D F.

PROBLEMA.

Datis duarum quarumvis quantitatum ſumma et differentia, ipſas quantitates invenire.



Si ad ſemiſummam addatur ſemidifferentia, aggregatum erit æquale majori; ſi autem a ſemiſumma ſubducatur ſemidifferentia, reſiduum erit æquale minori. Sint enim A B B C duæ quantitates; et capiatur A D = B C. Fiet D B differentia. Quarum ſumma eſt A C, quæ biſecta in E dat A E vel E C ſemiſummam et D E vel E B ſemidifferentiam. Porro eſt A B = A E + E B = ſemiſummæ + ſemidifferentia, et B C = C E − E B = ſemiſummæ − ſemidifferentia.

IN quovis triangulo plano datis duobus angulis, datur tertius qui eſt ſummæ duorum reliquorum complementum ad duos rectos.

In triangulo autem rectangulo dato alterutro angulo acuto, datur reliquus, qui eſt dati complementum ad rectum.

Datis autem duobus trianguli rectanguli lateribus, ut inveniatur reliquum non opus eſt canone, ſed perficitur ope Prop. primæ hujus.

Trianguli rectanguli ſolutiones Trigonometricæ ſunt, quæ ſequuntur.

	Datis.	Quær.	Fiat.
1	A B B C cruribus.	Anguli.	A B : B C : : R. : T. anguli A, cujus complementum eſt angulus C.
2	A B A C crure et hypoten.	Anguli.	A C : A B : : R. : S. C, cujus complementum eſt angulus A.
3	A B et A crure et angulo.	B C crus alterum.	R. : T. A : : A B : B C.
4	A B et C crure et angulo.	A C hypotenuſa.	S. C : R. : : A B : A C.

In triangulis obliquangulis.

	Datis.	Quær.	Fiat.
1	A, B, C, et A B angulis et latere.	B C et A C latera.	S. C : S. A : : A B : B C. Item S. C : S. B : : A B : A C; datis duobus angulis datur tertius, unde caſus cum dantur duo anguli et latus; reliqua quærantur, recidit in hunc caſum.
2	A, B, C, omnibus angulis.	A B, A C, B C omnia latera.	S. C : S. A : : A B : B C. Et S. C : S. B : : A B : A C. Unde datis angulis invenire licet proportiones laterum, at non ipſa latera, niſi ipſorum unum prius innoteſcat.

A B : B C,

	Datis.	Quær.	Fiat.
3	A B : B C, et C duobus lateribus et angulo uni oppoſito.	A et B anguli.	A B : B C : S. C : S. A, qui proinde inveniatur. Sed quia idem eſt ſinus anguli et ejus complementi ad duos rectos, prænoſcenda eſt anguli A ſpecies.
4	A B, B C, et B lateribus duobus et angulo interjecto.	Anguli A et C.	B C + A B : B C − A B : : T. $\frac{A+C}{2}$: T. $\frac{A-C}{2}$ unde datur differentia angulorum A et C, quorum ſumma quoque eſt nota; et proinde per *Problema poſt Prop.* xiv. dabuntur ipſi anguli.
5	A B, B C, A C omnibus lateribus.	Anguli.	Demiſſo a vertice in baſim perpendiculo, quærantur ſegmenta baſis per Prop. xiv. Fiat ſcil. B C : A C + A B : : A C − A B : D C − D B, et ex hac analogia dabuntur B D. D C. et proinde per reſolutionem triangulorum rectangulorum A B D A D C dabuntur anguli.

TRIGONOMETRIÆ

SPHÆRICÆ

ELEMENTA.

DEFINITIONES.

Definitio I.

SPHÆRÆ poli ſunt duo puncta in ſuperficie ſphærica, quæ ſunt axis extrema.

II.

Polus circuli in ſphæra eſt punctum in ſuperficie ſphæræ, a quo omnes rectæ lineæ ad circuli circumferentiam tendentes, ſunt inter ſe æquales.

III.

Circulus in ſphæra maximus eſt, cujus planum tranſit per ſphæræ centrum, et cujus centrum idem eſt cum centro ſphæræ.

IV.

Triangulum ſphæricum eſt figura comprehenſa ſub arcubus trium maximorum in ſphæra circulorum.

V.

Angulus ſphæricus eſt is qui in ſuperficie ſphærica continetur ſub duobus arcubus maximorum circulorum; qui æqualis eſt inclinationi planorum iſtorum circulorum.

PROP. I.

Circuli maximi A C B A F B *ſe bifariam ſecant.*

Cum enim circuli habent idem centrum, communis eorum ſectio erit utriuſque circuli diameter, quæ eos bifariam ſecabit.

Cor. Hinc in ſuperficie ſphæræ, duo maximorum circulorum arcus ſemicirculis minores, ſpatium non comprehendunt, non enim poſſunt, niſi in duobus punctis ſemicirculo oppoſitis, ſibi invicem occurrere.

PROP. II.

Si a polo C *circuli cujuſvis* A F B, *ducatur ad ejus centrum recta* C D, *ea ad planum iſtius circuli perpendicularis erit.*

In circulo A F B ducantur diametri quævis E F G H; et quoniam in triangulis C D F C D E, ſunt C D D F æquales C D D E, et baſis C F æqualis baſi C E (per Def. 2.) erit (per 8. El. I.) angulus C D F = angulo C D E; ac proinde uterque rectus erit, ſimiliter demonſtrabitur, angulos C D G C D H eſſe rectos; unde (per 4. El. XI.) erit C D perpendicularis ad planum circuli A F E. *Q. E. D.*

Cor. 1. Circulus maximus diſtat a polo ſuo intervallo quadrantis; nam ob angulos C D G C D F rectos, erunt ipſorum menſuræ, ſc. arcus C G C F quadrantes.

Cor. 2. Circuli maximi per polum alterius circuli tranſeuntes cum ipſo faciunt angulos rectos; et viciſſim, ſi cum altero circulo faciunt angulos rectos; tranſibunt per polum alterius iſtius circuli; nam per rectam D C eos tranſire neceſſe eſt.

PROP. III.

Si polo A *deſcribatur maximus circulus* E C F, *arcus* C F *interceptus inter* A C *&* A F, *eſt menſura anguli* C A F *vel* C D F.

Per *Corol.* 1. præcedentis, ſunt arcus A C A F quadrantes, ac proinde anguli A D C A D F ſunt recti, quare (per Defin. 6. El. XI.) angulus C D F (cujus menſura

eſt

eſt arcus C F) æqualis eſt inclinationi planorum A C B A F B, æqualis quoque angulo ſphærico C A F vel C B F. *Q. E. D.*

Cor. 1. Si arcus A C A F ſunt quadrantes, erit A polus circuli per puncta C et F tranſeuntis, eſt enim A D ad planum F D C normalis, (per 4. El. XI.)

Cor. 2. Anguli ad verticem ſunt æquales, uterque enim eſt æqualis inclinationi circulorum. Item anguli qui ſunt deinceps ſunt æquales duobus rectis.

PROP. IV.

Triangula erunt æqualia et congrua, ſi duo latera habeant duobus lateribus æqualia, et angulos æqualibus lateribus comprehenſos etiam æquales.

PROP. V.

Item triangula erunt æqualia et congrua, ſi latus cum angulis adjacentibus in uno triangulo ſit æquale lateri cum angulis adjacentibus in altero triangulo.

PROP. VI.

Triangula æquilatera ſunt etiam æquiangula.

PROP. VII.

In triangulis Iſoſcelibus, anguli ad baſim ſunt æquales.

PROP. VIII.

Si anguli ad baſim fuerint æquales, erit triangulum Iſoſceles.

Eodem modo demonſtrantur quinque propoſitiones præcedentes ut in triangulis planis.

PROP. IX.

Quælibet duo trianguli latera reliquo ſunt majora.

Nam arcus circuli maximi, inter duo quælibet in ſuperficie ſphæræ puncta, eſt via breviſſima.

PROP.

PROP. X.

Quodlibet trianguli latus minus eſt ſemicirculo.

Producantur trianguli A B C latera A C A B, donec conveniunt in D, erit arcus A C D ſemicirculus, qui major eſt quam A C.

PROP. XI.

Trianguli latera ſunt circulo minora.

Eſt enim D B + D C major quam B C, (per Prop. ix.) et utrinque addendo B A + A C, erit D B A + D C A, hoc eſt, circulus major quam A B + B C + A C, qui ſunt tria latera trianguli A B C.

PROP. XII.

In triangulo A B C, *major angulus* A *majori lateri ſubtenditur.*

Fiat angulus B A D = angulo B, et erit A D = B D (per viii. hujus) unde B D C = D A + D C, et hi arcus majores ſunt quam A C, eſt itaque latus B C, quod ſubtendit angulum B A C, majus quam A C, quod ſubtendit angulum B.

PROP. XIII.

In quolibet triangulo A B C, *ſi ſumma crurum* A B B C *ſit major æqualis vel minor ſemicirculo; internus angulus ad baſim* A C *erit major æqualis aut minor externo et oppoſito* B C D, *ideoque ſumma angulorum* A *et* A C B *major erit, aut æqualis, aut minor duobus rectis.*

Vide fig. Prop. 10.

Sit primo A B + B C = ſemicirculo = A D, erit B C = B D; et anguli B C D et D æquales, (per viii. hujus) unde et angulus B C D erit = angulo A.

Sit ſecundo A B + B C majores quam A B D, erit B C major quam B D; unde et angulus D, hoc eſt angulus A, major erit angulo B C D (per xii. hujus.) Similiter oſtendetur, ſi A B + B C ſint ſimul minores ſemicirculo, fore angulum A minorem angulo B C D. Et quoniam anguli B C D et B C A ſunt = duobus rectis; ſi angu-

lus

lus A fit major B C D, erit A et B C A majores duobus rectis. Si A fit = B C D, erit A et B C A æquales duobus rectis. Si vero A fit minor quam B C D, erunt A et B C A minores duobus rectis. *Q. E. D.*

PROP. XIV.

In quolibet triangulo G H D, *laterum poli, ductis circulis maximis, conſtituunt aliud triangulum* X M N, *quod ſupplementum eſt trianguli* G H D, *nempe latera* N X X M *et* N M *erunt ſupplementa ad ſemicirculos arcuum qui ſunt menſuræ angulorum* D, G, H. *Quinetiam menſuræ angulorum* M, X, N, *erunt ſupplementa ad ſemicirculos, laterum* G H, G D, *et* H D.

Polis G, H, D, deſcribantur maximi circuli X C A M, T M N O, X K B N. Et quia G eſt polus circuli X C A M, erit G M = quadranti, (per Cor. I. Prop. ii.) et ob H polum circuli T M O, erit H M quoque quadrans; quare (per Corol. I. Prop. iii.) erit M polus circuli G H. Similiter quia D eſt polus circuli X B N, et H polus circuli T M N, erunt arcus D N H N quadrantes; ac proinde (per Cor. I. Prop. iii.) N erit polus circuli H D. Et eadem ratione, ob G X D X quadrantes, erit X polus circuli G D. Hiſce præmiſſis.

Quoniam eſt N K = quadranti, (Cor. I. Prop. ii.) et X B = quadranti, erunt N K + X B, hoc eſt N X + K B = duobus quadrantibus ſeu ſemicirculo; adeoque eſt N X ſupplementum arcus K B ſeu menſuræ anguli H D G ad ſemicirculum. Similiter quia eſt M C = quadranti, et X A = quadranti; erunt M C + X A, hoc eſt, X M + A C = duobus quadrantibus ſeu ſemicirculo, et proinde X M eſt ſupplementum arcus A C, qui eſt menſura anguli H G D. Quinetiam, ob M O, N T quadrantes, erunt M O + N T = O T + N M = ſemicirculo. Itaque eſt N M ſupplementum ad ſemicirculum arcus O T, ſeu menſuræ anguli G H D. *Q. E. D.*

Præterea quia D K, H T ſunt quadrantes, erunt D K + H T ſeu K T + H D æquales duobus quadrantibus, ſeu ſemicirculo. Eſt ergo K T, ſeu menſura anguli X N M, ſupplementum lateris H D ad ſemicirculum. Nec diſſimili methodo oſtendetur O C menſuram anguli X M N eſſe

esse supplementum lateris G H. Et B A mensuram anguli X esse supplementum lateris G D. *Q. E. D.*

PROP. XV.

Triangula æquiangula sunt etiam æquilatera.

Nam eorum supplementa sunt æquilatera, (per xiv. hujus) ergo et æquiangula, quare et ipsa sunt æquilatera, per Prop. xiv. partem secundam.

PROP. XVI.

Trianguli tres anguli sunt majores duobus rectis, et minores sex rectis.

Vide fig. Prop. 14.

Nam tres mensuræ angulorum G, H, D, una cum tribus lateribus trianguli X N M faciunt tres semicirculos, (per xiv. hujus) sed tria latera trianguli X N M minora sunt duobus semicirculis, (per xi. hujus) quare tres mensuræ angulorum G H D majores sunt semicirculo, et proinde anguli G H D majores erunt duobus rectis.

Propositionis secunda pars patet, nam in quolibet triangulo, externi et interni anguli simul tantum faciunt sex rectos, unde interni sunt minores quam sex recti.

PROP. XVII.

Si a puncto R *quod circuli* A F B E *polus non est, in circumferentiam cadant arcus maximorum circulorum* R A R B R G R V, *maximus est* R A, *qui per ejus polum* C *incedit, reliquus vero minimus; cæteri, prout a maximo recedunt minores sunt, faciuntque cum priore circulo* A F B *angulum obtusum ex parte maximi arcus.*

Vide fig. Prop. 1.

Quia C est polus circuli A F B, erunt C D et huic parallela R S perpendiculares ad planum A F B; ductis autem S A S G S V; erit (per 7. El. III.) S A major quam S G, et S G major quam S V. Unde in triangulis rectangulis planis R S A R S G R S V, erunt R S q + S A q, seu R A q majora quam R S q + S A q seu R G q, et proinde R A major erit R G; et arcus R A major arcu R G. Similiter erunt R S q + S G q, seu R G q majora quam

quam R S q + S V q ſeu R V q; et proinde R G major R V, et arcus R G major arcu R V.

2*do*. Eſt angulus R G A major angulo C G A qui rectus eſt, (per Corol. Prop. iii.) Et angulus R V A major angulo C V A qui quoque rectus eſt, quare anguli R G A R V A ſunt obtuſi.

PROP. XVIII.

In triangulo rectangulo ad A, *crura angulum rectum continentia ſunt ejuſdem affectionis cum angulis oppoſitis, hoc eſt, ſi crura ſint majora aut minora quadrantibus, anguli illis oppoſiti erunt majores aut minores rectis angulis.*

Vide fig. Prop. I.

Nam ſi A C ſit quadrans, C erit polus circuli A F B, et anguli A G C vel A V C erunt recti. Si crus A R ſit majus quadrante, erit angulus A G R major recto (per xvii. hujus.) Si crus ſit minus quadrante ut A X, angulus A G X erit minor recto.

PROP. XIX.

Si duo crura trianguli rectanguli (et conſequenter anguli) ſint ejuſdem affectionis, id eſt, utrumque vel majus vel minus quadrante, hypotenuſa erit minor quadrante.

Vide fig. Prop. I.

In triangulo A R V vel B R V, ſit F polus cruris A R, et erit R F quadrans, qui major eſt quam R V (per xvii. hujus.)

PROP. XX.

Si ſint diverſæ affectionis, hypotenuſa erit major quadrante.

Nam in triangulo A R G, eſt R G major quam R F qui eſt quadrans.

PROP.

PROP. XXI.

Si hypotenuſa ſit major vel minor quadrante, crura anguli recti, ideoque et anguli oppoſiti ſunt ejuſdem aut diverſæ affectionis.

Hæc propoſitio eſt priorum converſa; et facile ex iiſdem ſequitur.

PROP. XXII.

In quovis triangulo A B C, *ſi anguli* B *et* C *ad baſim ſunt ejuſdem affectionis, perpendicularis* A P *cadet intra triangulum; ſi ſint diverſæ affectionis, perpendicularis cadet extra triangulum.*

In primo caſu ſi perpendicularis non cadat intra, cadet extra triangulum, (ut in fig. 2.) Tum in triangulo A B P, eſt A P ejuſdem affectionis cum angulo B; et ſimiliter in triangulo A C P, eſt A P ejuſdem affectionis cum angulo A C P; ergo cum A B C et A C P ſunt ejuſdem affectionis, erunt anguli A B C et A C B diverſæ affectionis; quod eſt contra hypotheſim.

In 2*do* caſu, ſi perpendicularis non cadat extra, cadet intra, (ut in fig. 1.) Et in triangulo A B P, eſt angulus B ejuſdem affectionis cum crure A P, et ſimiliter in triangulo A C P eſt angulus C ejuſdem affectionis cum A P, unde anguli B et C ſunt ejuſdem affectionis, quod eſt contra hypotheſim.

PROP. XXIII.

In triangulis B A C B H E *rectangulis ad* A et H, *ſi idem fuerit angulus acutus* B *ad baſim* B A *vel* B H, *ſinus hypotenuſarum erunt ſinubus arcuum perpendicularium proportionales.*

Nam rectæ C D E F perpendiculariter inſiſtentes eidem plano ſunt parallelæ. Item F R D P radio O B perpendiculares, ſunt quoque parallelæ; unde et plana triangulorum E F R C D P ſunt parallela (per 15. El. XI.) Quare et C P E R horum planorum communes ſectiones cum plano per B E C O tranſeunte parallelæ erunt (per 16. El. XI.) Triangula igitur C D P E F R æquiangula erunt.

erunt. Quare C P ſinus hypotenuſæ B C eſt ad C D ſinum arcus perpendicularis C A; ut E R ſinus hypotenuſæ B E eſt ad E F ſinum arcus perpendicularis E H. *Q. E. D.*

PROP. XXIV.

Iiſdem poſitis, A Q H K *ſinus baſium, tangentibus* I A G H *arcuum perpendicularium, ſunt proportionales.*

Nam ſimiliter ut in præcedente propoſitione, oſtendetur triangula Q A I K H G eſſe æquiangula; unde Q A : A I : : K H : H G.

PROP. XXV.

In triangulo A B C *rectangulo ad* A. *Ut coſinus anguli* B *exiſtentis ad baſim* B A *ad ſinum anguli verticalis* A C B, *ita coſinus arcus perpendicularis ad radium.*

Præparatio. Producantur latera B A B C C A ita ut B E B F C I C H ſint quadrantes, polis B et C ducantur circuli maximi E F D G I H G. et erunt anguli ad E, F, I et H recti. Quare D eſt polus B A E (per Cor. 2. Prop. ii. hujus) et G polus I F C B, erit etiam A E = complemento arcus B A. Item F E menſura anguli B = G D, et D F eorum complementum. Erit quoque B C = F I = menſuræ anguli G, et C F eorum complementum. Item eſt C A = H D, et D C utriuſque complementum. Hiſce præmiſſis, in triangulis H I C D C F rectangulis ad I et F et habentibus eundem angulum C acutum, ob B A minorem quadrante, erit S. D F : S. H I : : S. D C : S. H C id eſt, coſinus anguli B eſt ad ſinum anguli verticalis B C A ut coſinus C A ad radium. *Q. E. D.*

PROP. XXVI.

Coſinus baſis : coſin. hypotenuſæ : : R : coſ. perpendicularis.

Nam in triangulis A E D C F D rectangulis ad E et F; habentibus eundem angulum D acutum: ob A E quadrante minorem, eſt S. E A : S. C F : : S. D A : S. D C. *Q. E. D.*

PROP.

PROP. XXVII.

S. baſeos : R : : T. *perpendicularis* : T. *anguli ad baſim.*

Nam in triangulis B A C B E F rectangulis ad A et E et habentibus eundem angulum B acutum, ob A C minorem quadrante, S. B A : S. B E : : T. A C : T. E F. *Q. E. D.*

PROP. XXVIII.

cof. anguli verticalis : R. : : T. *perpendicularis* : T. *hypotenuſæ.*

In triangulis G I F G H D rectangulis ad I et H, et habentibus eundem angulum G acutum, ob H D minorem H C ſeu quadrante, eſt S. G H : S. G I : : T. H D : T. I F.

PROP. XXIX.

S. *hypotenuſæ* : R. : : S. *perpendicularis* : S. *anguli ad baſim.*

In triangulis præcedentibus, eſt S. I F : S. G F : : S. H D : S. G D.

PROP. XXX.

Radius : *cof. hypotenuſæ* : : T. *anguli verticalis* : *cot. anguli ad baſim.*

In triangulis H I C D F C rectangulis ad I et F, et habentibus eundem angulum C acutum, ob D F minorem quadrante, eſt S. C I : S. C F : : T. H I : T. D F. hoc eſt, R. : cof. B C : : tang. C : cot. anguli B.

Propoſitiones ſex præcedentes ad omnes caſus triangulorum rectangulorum reſolvendos ſufficiunt, ſequuntur illi numero ſedecim cum ſuis analogiis ex hiſce deductis.

Datis

	Datis præter ang. rectum.	Quær.		
1	A C et C	B	R : cof. C A : : S. C : cof. B ejuſdem ſpeciei cum C A.	per 25 inverſe.
2	A C et B	C	cof. C A : R. : : cof. B : S. C ambigui.	per 25.
3	B et C	A C	S. C : cof. B : : R. : cof. C A ejuſdem ſpeciei cum ang. B.	per 25 et 18.
4	B A C A	B C	R. : cof. B A : : cof. A C : cof. B C. Si B A A C fuerint ejuſdem affectionis nec quadrantes, erit B C minor quadrante; ſi diverſæ, erit B C quadrante major.	per 26 et 19 20.
5	B A B C	A C	cof. B A : R. : : cof. B C : cof. C A. Si B C ſit major aut minor quadrante, B A et C A erunt ejuſdem aut diverſæ affectionis, ſed datur B A ejuſque ſpecies, ergo.	per 26 et 21.
6	B A C A	B	S. B A : R. : : T. C A : T B ejuſdem affectionis cum latere oppoſito C A.	per 27 et 18.
7	B A B	A C	R. : S. B A : : T. B : T. A C ejuſdem ſpeciei cum B.	per 27 et 18.
8	A C B	B A	T. B : R. : : T. C A : S. B A ambigui.	per 27.
9	B C C	A C	R. : cof. C : : T. B C : T. C A. Si B C ſit major aut minor quadrante, anguli C et B ſunt ejuſdem aut diverſæ affectionis, quare data ſpecie ang. B dabitur A C.	per 28 et 21.

	Datis præter ang. rectum.	Quær.		
10	A C C	B C	cos. C : R. : : T. A C : T. B C. prout ang. C et A C fuerint ejuſdem aut diverſæ affectionis, B C erit minor aut major quadrante.	per 18, 19, 20.
11	B C A C	C	T. B C : R. : : T. C A : cos. C. Si B C fuerit major aut minor quadrante, C A et B A et proinde anguli C et B erunt ejuſdem aut diverſæ affectionis, ſed datur ſpecies C A, ergo dabitur ſpecies anguli C.	per 28, 21.
12	B C B	A C	R. : S. B C : : S. B : S. A C ejuſdem ſpeciei cum B.	per 29 et 18.
13	A C B	B C	S. B : S. A C : : R. : S. B C ambigui.	per 29.
14	B C A C	B	S. B C : R. : : S. A C : S. B ejuſdem ſpeciei cum C A.	per 29.
15	B C	B C	T. C : R. : : cot. B : cos. B C. prout anguli B et C ejuſdem aut diverſæ affectionis fuerint, erit B C minor aut major quadrante.	per 30, 19, 20.
16	B C C	B	R. : cos. B C : : T. C : cot. B. prout B C fuerit minor aut major quadrante; anguli C et B erunt ejuſdem aut diverſæ affectionis, ſed datur ſpecies anguli C, quare dabitur ſpecies anguli B.	per 30, 21.

De Reſolutione Triangulorum Rectangulorum Sphæricorum, per quinque partes circulares.

PERPENSIS analogiis, quibus triangula ſphærica rectangula ſolvuntur, Dominus Neperus, nobilis ille Logarithmorum Inventor, duas excogitavit regulas memoria facile retinendas, quarum ope omnes ſedecim caſus reſolvi poſſunt; nam cum in hiſce triangulis, præter angulum rectum, ſint tria latera et duo anguli, latera angulum rectum comprehendentia, hypotenuſæ autem et reliquorum angulorum complementa, vocavit Neperus *partes circulares*. Et cum datæ ſunt duæ quælibet partes, et quæritur tertia, harum trium una, quæ dicitur *pars media*, vel adjacet duobus reliquis partibus, quæ itaque vocantur *extremæ adjacentes*; vel neutri adjacet, in quo caſu, dicuntur *extremæ oppoſitæ*; ſic ſi complementum anguli B ponatur pars media, crus A B et complementum hypotenuſæ B C ſunt partes extremæ adjacentes; at complementum anguli C, et latus A C ſunt extremæ oppoſitæ. Item poſito complemento hypotenuſæ B C parte media, complementa angulorum B et C ſunt extremæ adjacentes; et A B A C crura ſunt extremæ oppoſitæ. Sic etiam poſito crure A B parte media, complementum anguli B, et A C ſunt extremæ adjacentes; nam angulus rectus A non intercipit adjacentiam, quia non eſt pars circularis. At eidem parti mediæ complementum anguli C et complementum hypotenuſæ B C ſunt extremæ oppoſitæ. Hiſce præmiſſis.

REGULA PRIMA.

In triangulo rectangulo ſphærico, rectangulum ſub radio et ſinu partis mediæ, æquale eſt rectangulo ſub tangentibus partium adjacentium.

REGULA SECUNDA.

Rectangulum ſub radio et ſinu partis mediæ, æquale eſt rectangulo ſub coſinubus partium oppoſitarum.

Utriuſque regulæ tres ſunt caſus. Nam pars media vel poteſt eſſe complementum anguli B vel C, vel complementum hypotenuſæ B C; vel denique unum ex cruribus, ſcil. A B vel A C.

Caſus 1. Sit complementum anguli C pars media; et erunt A C et complementum hypotenuſæ B C extremæ adjacentes. Per Prop. xxviii. Eſt ut coſinus anguli verticalis C ad radium, ita tangens C A ad tangentem hypotenuſæ B C. Permutando erit cof. C : T. C A :: R. : T. B C. Sed ut notum eſt, R. : T. B C :: cot. B C : R. quare cof. C : T. A C :: cot. B C : R.; unde R. × cof. C = T. A C × cot. B C.

Eidem complemento anguli C parti mediæ, extremæ oppoſitæ ſunt complementum anguli B et A B, et (per Prop. xxv.) coſinus anguli C eſt ad ſinum anguli C D F ut coſinus D F ad radium, eſt vero ſinus C D F = S. A B = cof. B A, et cof. D F = S. E F = S. ang. B. Unde erit cof. C : cof. B A :: S. B : R. et R. × cof. C = cof. B A × S. B, hoc eſt, radius ductus in ſinum partis mediæ, æquatur rectangulo ſub coſinubus extremarum oppoſitarum.

Caſus 2. Sit complementum hypotenuſæ B C pars media, et complementa angulorum B et C erunt extremæ adjacentes. In triangulo D C F (per Prop. xxvii.) eſt S. C F : R. :: T. D F : T. C; unde permutando S. C F : T. D F :: (R. : T. C ::) cot. C : R. Eſt autem S. C F = cof. B C et T. D F = cot. B. quare eſt R. × cof. B C = cot. C × cot. B. Hoc eſt, radius ductus in ſinum partis mediæ æquatur producto ex tangentibus partium adjacentium extremarum.

Eidem parti mediæ, ſcil. complemento B C, adſunt extremæ oppoſitæ A B A C, et (per Prop. xxvi.) eſt cof. B A : cof. B C :: R. : cof. A C. Quare erit R. × cof. B C = cof. B A × cof. A C.

Caſus 3. Sit denique A B pars media, et erunt complementum anguli B et A C extremæ adjacentes, (et per Prop.

Prop. xxvii.) S. A B : R. : : T. C A : T B ; unde erit S. A B : T. C A : : (R. : T. B : :) cot. B : R. adeoque erit R × S. A B = T. C A × cot. B.

Præterea parti mediæ A B, complementum B C, et complementum anguli C ſunt extremæ oppoſitæ ; et in triangulo G H D (per Prop. xxv.) eſt cof. D : S. D G H : : cof. G H : R. Eſt vero cof. D = cof. A E = S. A B, et S. G = S. I F = S. B C. Item eſt cof. G H = S. H I = S. C ; quare erit S. A B : S. B C : : S. C : R. Et hinc R × S. A B = S. B C × S. C.

Itaque in omni caſu, rectangulum ſub radio et ſinu partis mediæ æquale erit tam rectangulo ſub coſinubus extremarum oppoſitarum, quam rectangulo ſub tangentibus extremarum adjacentium. Et proinde ſi æquationes illæ reſolvantur in analogias (per 16. Elem. VI.) ope regulæ proportionis, partes ignotæ innoteſcent. Et ſi pars quæſita ſit media, primus analogiæ terminus erit radius, ſecundum et tertium occupant locum tangentes vel coſinus partium extremarum. Si vero quæratur extremarum una, analogia incipi debet cum altera, atque radius ſinuſque partis mediæ, in mediis ponantur locis, ut quartum teneat pars quæſita.

IN triangulis ſphæricis obliquangulis B C D, demiſſo arcu perpendiculari A C, ab angulo C in baſim B D, (productam ſi opus fuerit,) ut duo fiant triangula B A C D A C, rectangula ; eorum ope reſolvi poſſunt plerique caſus triangulorum obliquangulorum.

PROP. XXXI.

Coſinus angulorum B *et* D *ad baſim* B D, *ſinubus angulorum verticalium* B C A D C A *ſunt proportionales.*

Nam cof. ang. B : S. B C A : : (cof. C A : R. : :) cof. D : S. D C A (per 25. hujus.)

PROP. XXXII.

Coſinus laterum B C D C *ſunt proportionales coſinubus baſium* B A D A.

Eſt enim cof. B C : cof. B A : : (cof. C A : R. : :) cof. D C : cof. D A. (per 26. hujus.)

PROP.

PROP. XXXIII.

Sinus baſium B A D A, *ſunt in reciproca proportione tangentium angulorum* B *et* D *ad baſim* B D.

Quia per 27. hujus eſt, S. B A : R. : : T. A C : T. anguli B. Item per eandem, inverſe R. : S. D A : : T. ang. D : T. A C. Erit ex æquo in perturbata ratione (per 23. El. V.) S. B A : S. D A : : T. ang. D : T. ang. B.

PROP. XXXIV.

Tangentes laterum B C D C *ſunt in reciproca proportione coſinuum angulorum verticalium* B C A, D C A.

Quia per 28. hujus permutando, Eſt

T. B C : R. : : T. C A : cof. B C A,

et per eandem R. : cof. D C A : T. D C : T. C A,

quare ex æquo in perturbata ratione eſt

T. B C : cof. D C A : : T. D C : cof. B C A.

PROP. XXXV.

Sinus laterum B C D C *ſinubus angulorum oppoſitorum* B *et* D *ſunt proportionales.*

Quia per 29. hujus S. B C : R. : : S. C A : S. ang. B, et per eandem inverſe R. : S. D C : : S. ang. D : S. C A; erit ex æquo in perturbata ratione S. B C : S. D C : : S. D : S. B.

PROP. XXXVI.

In triangulo quovis ſphærico A B C, C F X A E *vel* F M X A E, *rectangulum ſub ſinubus crurum* B C B A *eſt ad radii quadratum, ut* I L *ſeu* I A — L A *differentia ſinuum verſorum baſis* A C, *et differentiæ crurum* A M, *ad* G N *ſinum verſum anguli* B.

Polo B deſcribatur circulus maximus P N; ſintque B P B N quadrantes; et P N eſt menſura anguli B; eodem polo B per C deſcribatur circulus minor C F M; horum circulorum plana recta erunt plano B O N, (per 2. hujus) et P G C H perpendiculares in idem planum, cadent in communes ſectiones O N F M puta in G et H. Ducatur H I perpendicularis ad A O, et planum per C H H I perpendiculare erit plano A O B, unde A I perpendicularis

dicularis ad HI, erit perpendicularis ad rectam CI, (per Def. 4. El. XI.) eſt itaque AI ſinus verſus arcus AC, et AL ſinus verſus arcus AM = BM — BA = CB — BA. Triangula Iſoſcelia CFM PON ſunt æquiangula ob MF NO, item CF PO parallelas, (per 16. El. XI.) quare demiſſis perpendiculis CH PG in latera FM ON, ſimiliter diviſa erunt triangula; et erit FM : ON :: MH : GN. Itemque ob triangula AOE DIH DLM æquiangula erit AE : AO :: IL : MH, at oſtenſum eſt, eſſe FM : ON :: MH : GN, quare erit AE × FM ad AO × ON, ut IL × MH ad MH × GN, ſeu ut IL ad GN. Hoc eſt rectangulum ſub ſinubus crurum eſt ad quadratum radii ut differentia ſinuum verſorum baſis et differentiæ crurum BC BA ad ſinum verſum anguli B. *Q. E. D.*

PROP. XXXVII.

Differentia ſinuum verſorum duorum arcuum ducta in dimidium radii, æqualis eſt rectangulo ſub ſinu ſemiſummæ et ſinu ſemidifferentiæ eorundem arcuum.

Sint duo arcus BE BF, quorum differentia EF ſit biſecta in D, et erit BD ſemiſumma arcuum, et FD ſemidifferentia. Eſt GE = IL differentiæ ſinuum verſorum arcuum BE BF; item eſt FO ſinus ſemidifferentiæ arcuum. Ob æquiangula triangula CDK FEG; erit DK : GE :: (CD : FE ::) ½ CD : ½ FE. Unde eſt DK × ½ FE, ſeu DK × FO = GE × ½ CD = IL × ½ CD. *Q. E. D.*

PROP. XXXVIII.

Sinus verſus cujuſvis arcus, ductus in dimidium radii, æqualis eſt quadrato ſinus dimidii ejuſdem arcus.

Triangula CBM DEB ſunt æquiangula ob angulos ad M et E rectos, et angulum ad B communem. Quare eſt EB : BD :: BM : BC, erit itaque EB × BC = BM × BD, et EB × ½ BC = BM × ½ BD = BMq. *Q. E. D.*

PROP.

PROP. XXXIX.

In quolibet triangulo A B C, *cujus crura angulum* B *continentia ſint* B C A B, *et baſis* A C *eundem angulum ſubtendat; ſi capiatur* A M *arcus = differentiæ crurum =* B C − A B, *erit rectangulum ſub ſinubus crurum* B C B A *ad quadratum radii, ut rectangulum ſub ſinu arcus* $\frac{\text{A C} + \text{A M}}{2}$, *et ſinu arcus* $\frac{\text{A C} - \text{A M}}{2}$, *ad quadratum ſinus dimidii anguli* B.

Vide fig. Prop. 36. *hujus.*

Quoniam eſt rectangulum ſub ſinubus crurum A B B C ad quadratum radii, ut I L ad ſinum verſum anguli B, vel ut ½ R. × I L ad ½ R. ductum in ſinum verſum anguli B. (per Prop. xxxvi. hujus.) Eſt autem ½ R × I L = rectangulo ſub ſinubus arcuum $\frac{\text{A C} + \text{A M}}{2}$ et $\frac{\text{A C} - \text{A M}}{2}$. (per Prop. xxxvii. hujus.) Item eſt ½ R. ductus in ſinum verſum anguli B æqualis quadrato ſinus dimidii anguli B. Quare erit rectangulum ſub ſinubus crurum, ad radii quadratum, ut rectangulum ſub ſinubus arcuum $\frac{\text{A C} + \text{A M}}{2}$ et $\frac{\text{A C} - \text{A M}}{2}$ ad quadratum ſinus dimidii anguli B. *Q. E. D.*

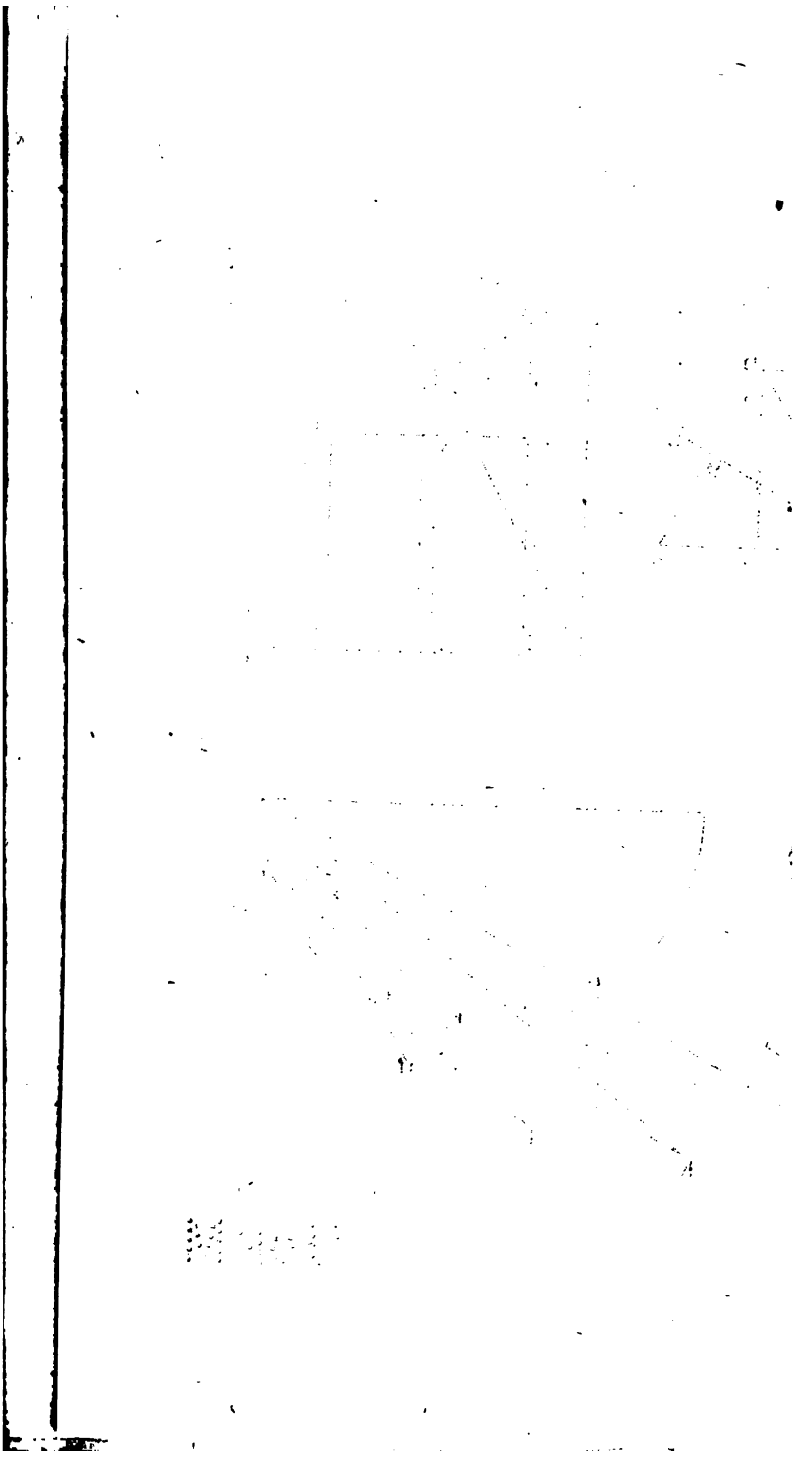
Sequuntur

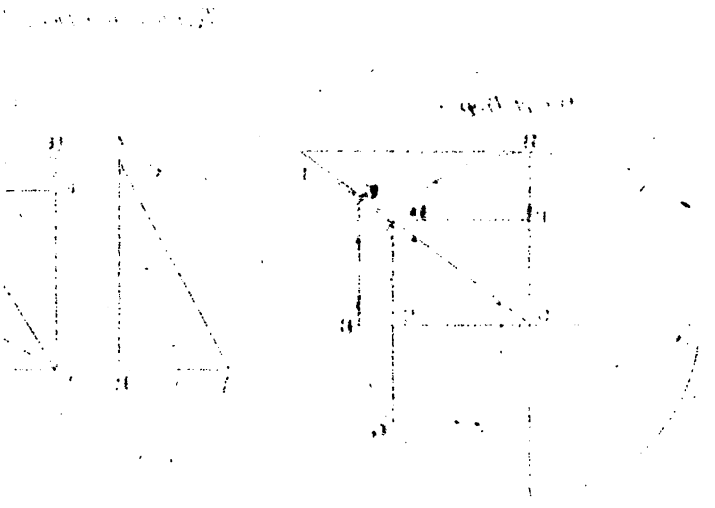
Sequuntur duodecim casus triangulorum sphæricorum obliquangulorum.

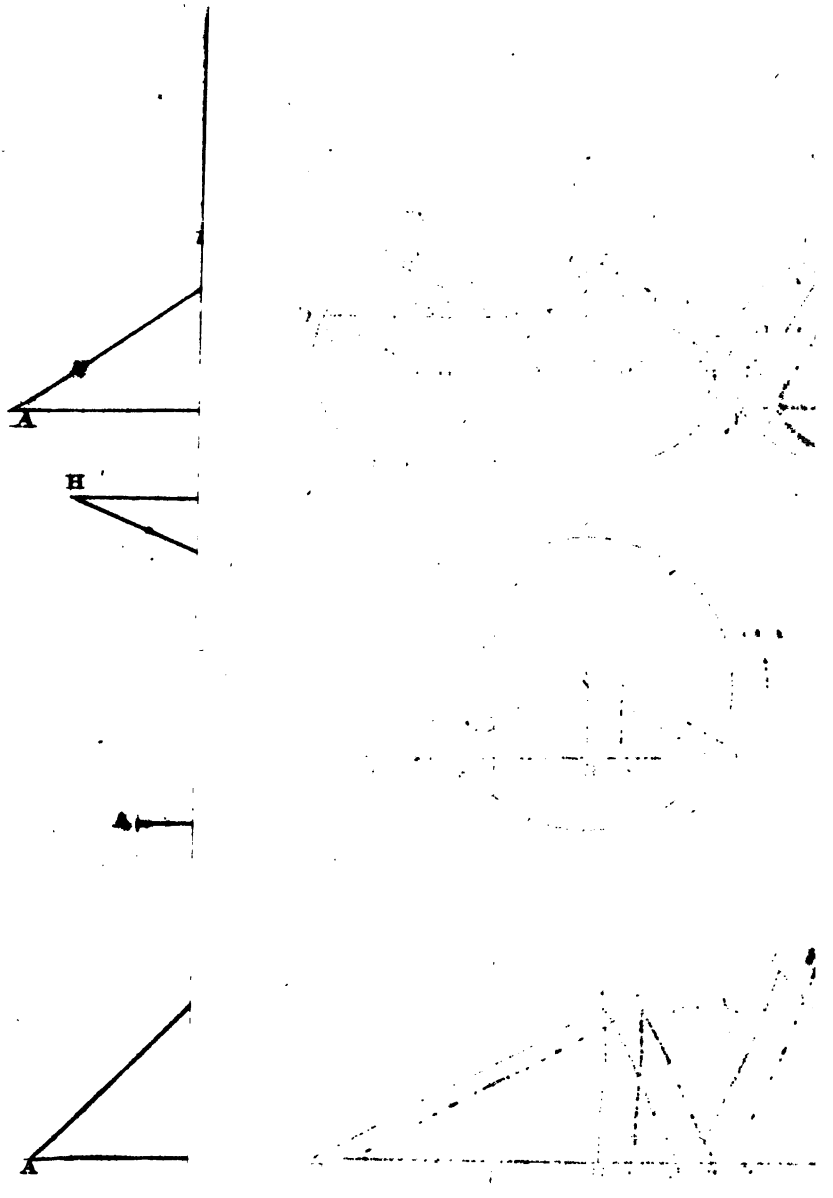
	Datis.	Quær.	Fiat. *Vide fig. Prop.* 31.
1	Ang. B, D, et B C.	Ang. C.	cos. B C : R. : : cot. B : T. B C A (per XXX. hujus). Item cos. B : S. B C A : : cos. D : S. D C A (per XXXI. hujus). Horum angulorum B C A D C A summa, si perpendicularis cadat intra triangulum, vel differentia, si extra cadat; erit = B C D. Num perpendicularis cadat intra vel extra, cognoscitur ex affectione angulorum B et D (per XXII. hujus), quod semel monuisse sufficiat.
2	Ang. B, C, et latere B C.	Ang. D.	cos. B C : R. : : cot. B : T. B C A (per XXX. hujus), et S. B C A ; S. D C A : : cos. B : cos. D (per XXXI. hujus). Si B C A fit minor B C D, angulus D erit ejusdem affectionis cum angulo B. Sin B C A fit major B C D, anguli B et D erunt affectionis diversæ per conversam Prop. XXII.
3	B C C D lateribus et ang. B.	B D latus.	R. : cos. B : : T. B C : T. B A (per XXVIII. hujus), et cos. B C : cos. B A : : cos. D C : cos. D A (per XXXII. hujus), horum B A D A summa vel differentia, prout perpendicularis cadit intra vel extra triangulum, est æqualis B D; quod cognosci nequit nisi cognita sit species alterius anguli D.
4	B C D B lateribus et ang. B.	C D latus.	R. : cos. B : : T. B C : T. B A (per XXVIII. hujus). Et cos. B A : cos. B C : : cos. D A : cos. D C (per XXXII. hujus). Prout D A similis est aut dissimilis C A vel ang. D B C, erit D C minor aut major quadrante (per XIX. et XX. hujus.)

	Datis.	Quær.	Fiat.
5	B, D, ang. et B C latere.	B D latus.	R. : cos. B : : T. B C : T. B A (per xxviii. hujus). Et T. D : T. B : : S. B A : S. D A (per xxxiii. hujus), quorum B A D A ſumma vel differentia = B D.
6	B C B D lateribus et ang. B.	Ang. D.	R. : cos. B : : T. B C : T. B A (per xxviii. hujus). Et S. D A : S. B A : : T. B : T. D (per xxxiii. hujus). Prout B D minor eſt aut major quam B A, angulus D ſimilis aut diſſimilis erit angulo B (per xxii. hujus).
7	B C D C lateribus et ang. B.	Ang. C.	cos. B C : R. : : cot. B : T. B C A (per xxx. hujus). Et T. D C : T. B C : : cos. B C A : cos. D C A (per xxxiv. hujus). Angulorum B C A D C A ſumma aut differentia, prout perpendicularis cadit intra vel extra triangulum, eſt æqualis angulo B C D.
8	B, C, ang. et B C latere.	D C latus.	cos. B C : R. : : cot. B : T. B C A (per xxx. hujus). Item cos. D C A : cos. B C A : T. B C : T. D C (per xxxiv. hujus). Si angulus D C A ſimilis ſit angulo B, (hoc eſt, ſi A D ſit ſimilis C A) erit D C minor quadrante. Si anguli D C A et B ſint diſſimiles, erit D C quadrante major, quod ſequitur (ex Prop. xviii. xix. et xx. hujus).
9	B C D C lateribus et aug. B.	D. ang.	S. C D : S. B : : S. B C : S. D, qui ambiguus eſt. Analogia ſequitur (ex Prop. xxxv. hujus.)
10	B D ang. et B C lat.	D C	S. D : S. B C : : S. B : S. D C, quod latus ambiguum eſt.

	Datis.	Quær.	Fiat.
11	A B B C C A omnibus lateribus, *vide fig. Prop.* XXXVI.	Ang. B.	Rectangulum ſub ſinubus crurum A B B C : quadratum radii : : rectangulum ſub ſinubus arcuum $\frac{AC+AM}{2}$ $\frac{AC-AM}{2}$: quadrato ſinus ½ ang. B. per Prop. XXXIX.
12	G H D omnibus ang.	G D latus.	*Vide fig. Prop.* 14. In triangulo X N M, eſt M N complementum anguli G H D ad ſemicirculum. X M complementum anguli G, et X N complementum anguli D; et angulus X complementum eſt lateris G D ad ſemicirculum. Quare mutatis angulis in latera, et lateribus in angulos; eadem eſt operatio quæ eſt in caſu 11. hujus, cum arcus et eorum complementa ad ſemicirculos habeant eoſdem ſinus.



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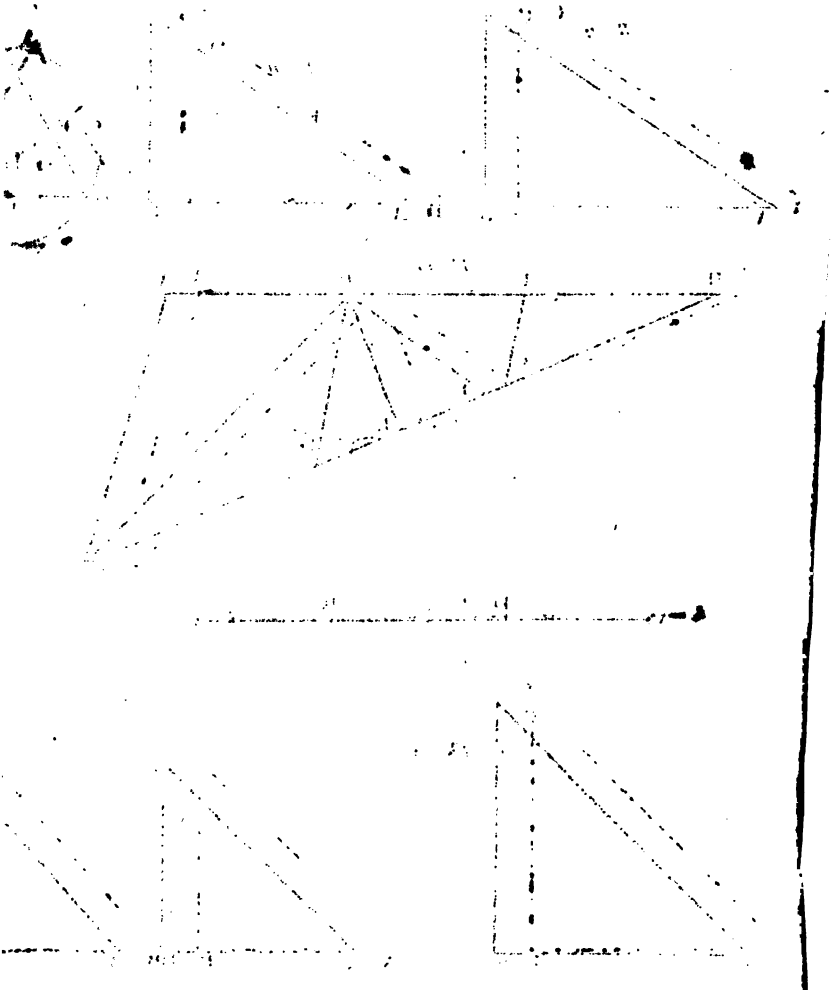
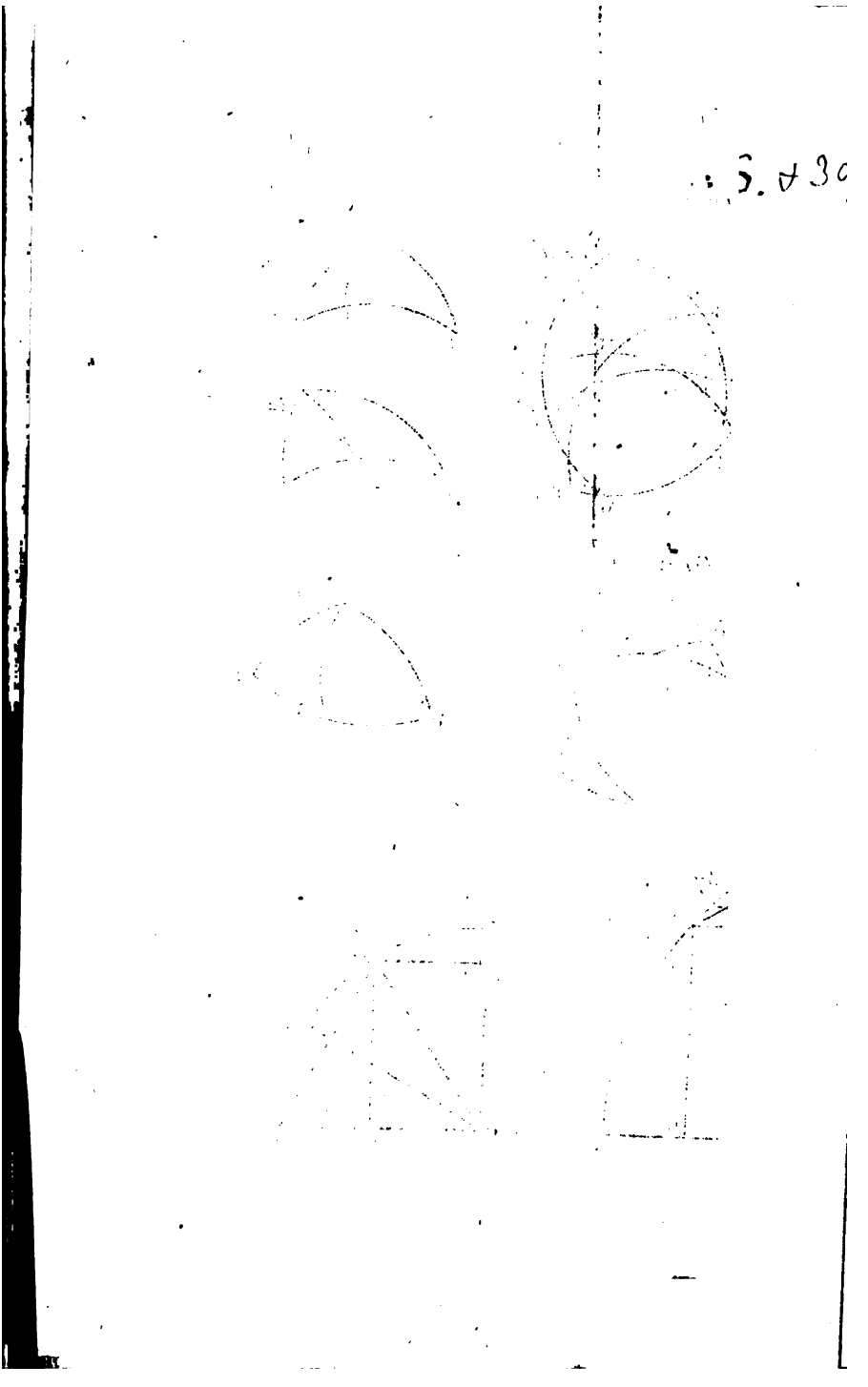
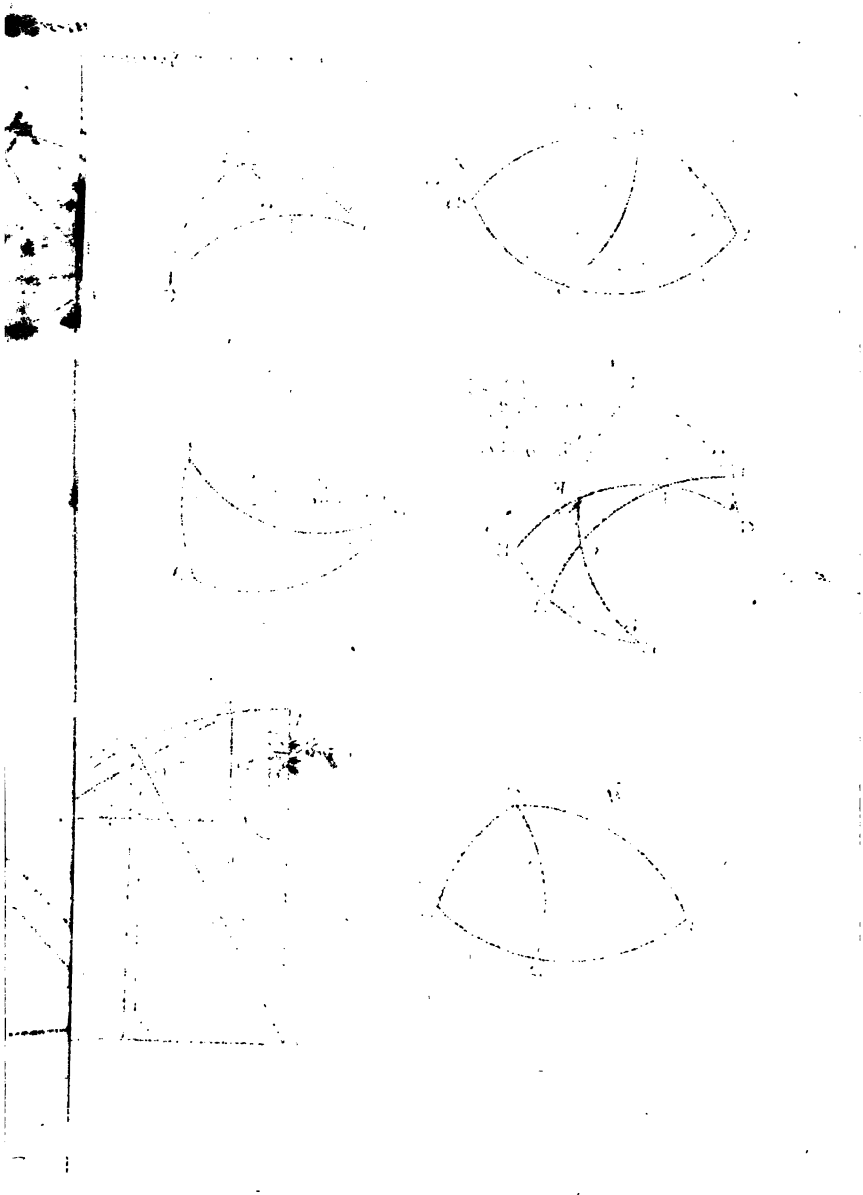
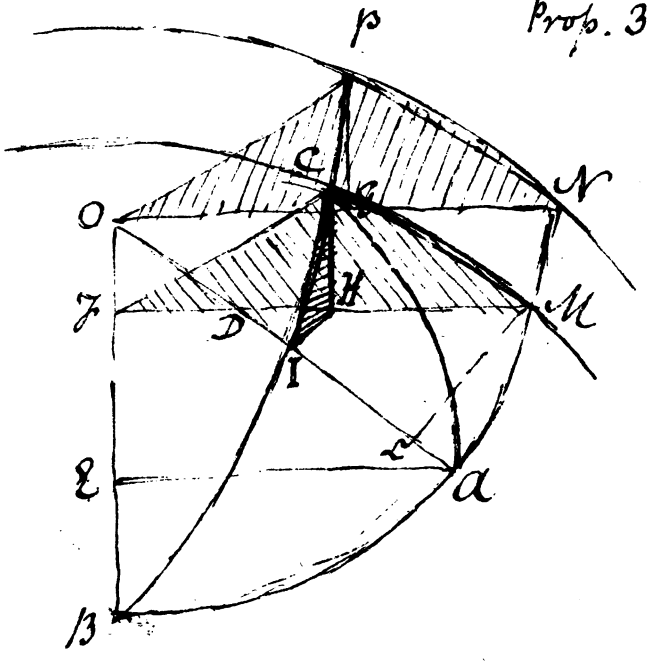
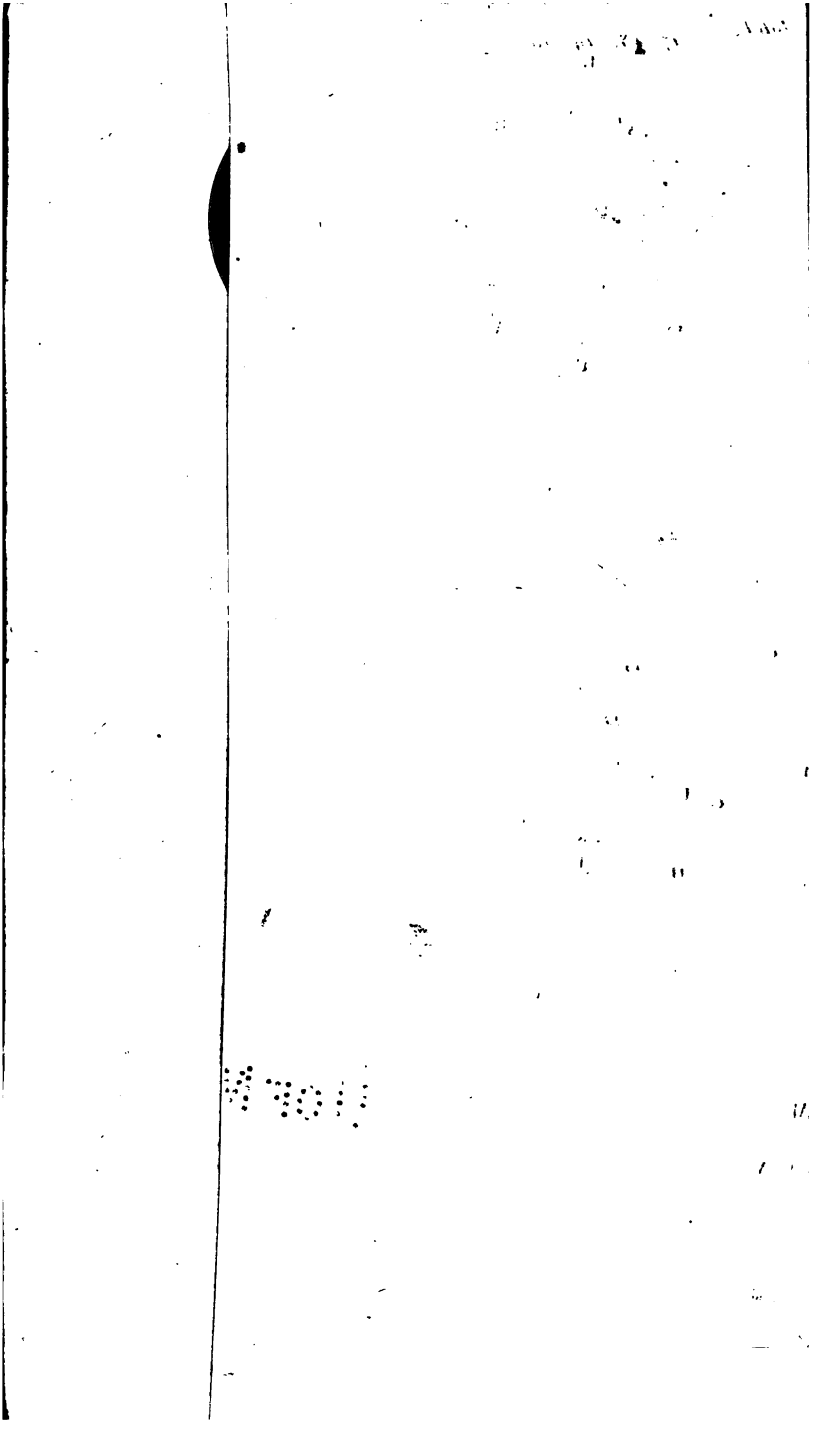






Prop. 36. & 3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